CBS (Confined Beta-Soft) calculations in the rare-earth mass region

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Presentation Outline

1. Area of Interest & Motivation

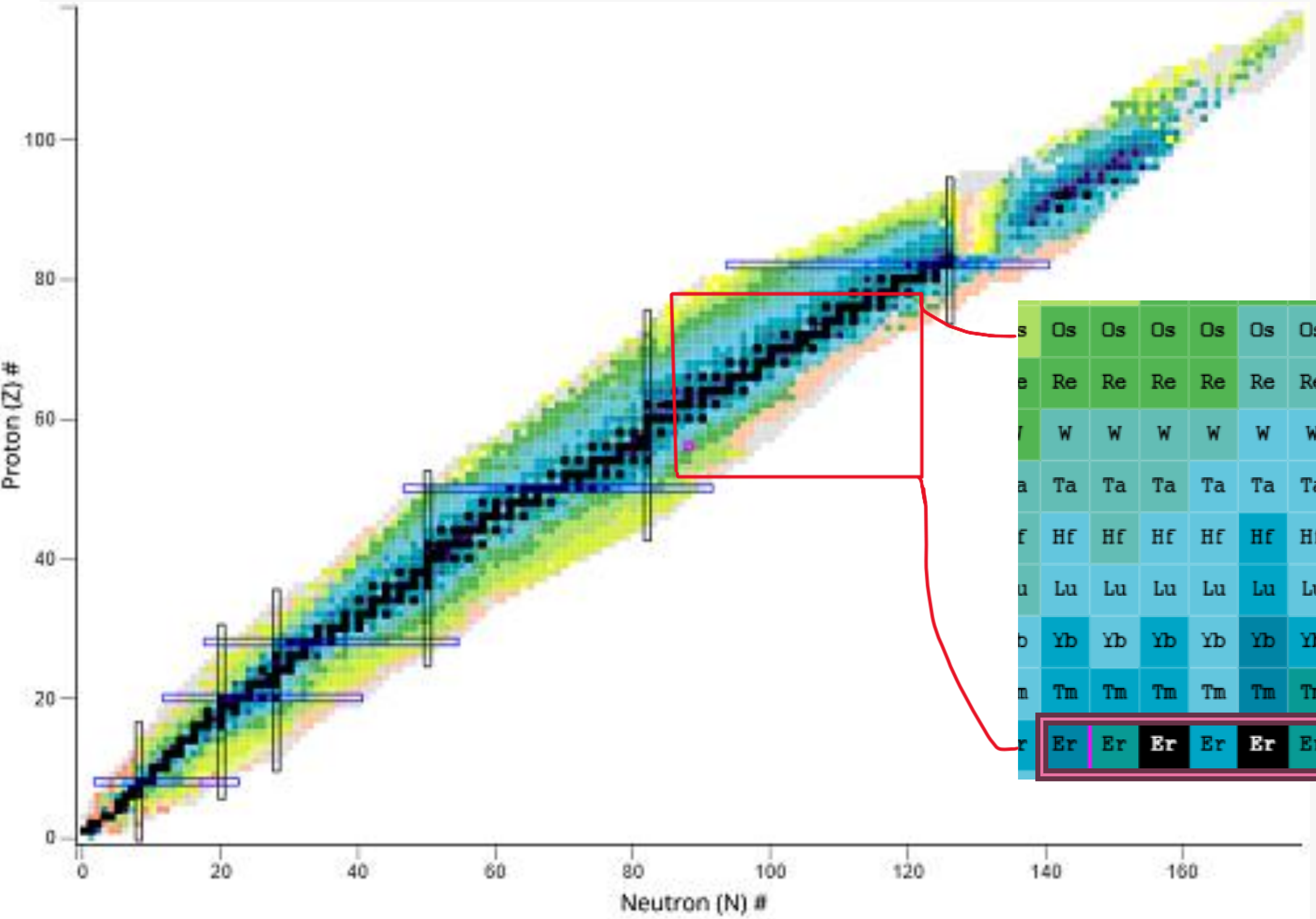
2. Theoretical Framework (Related Quantities & Theoretical Models)

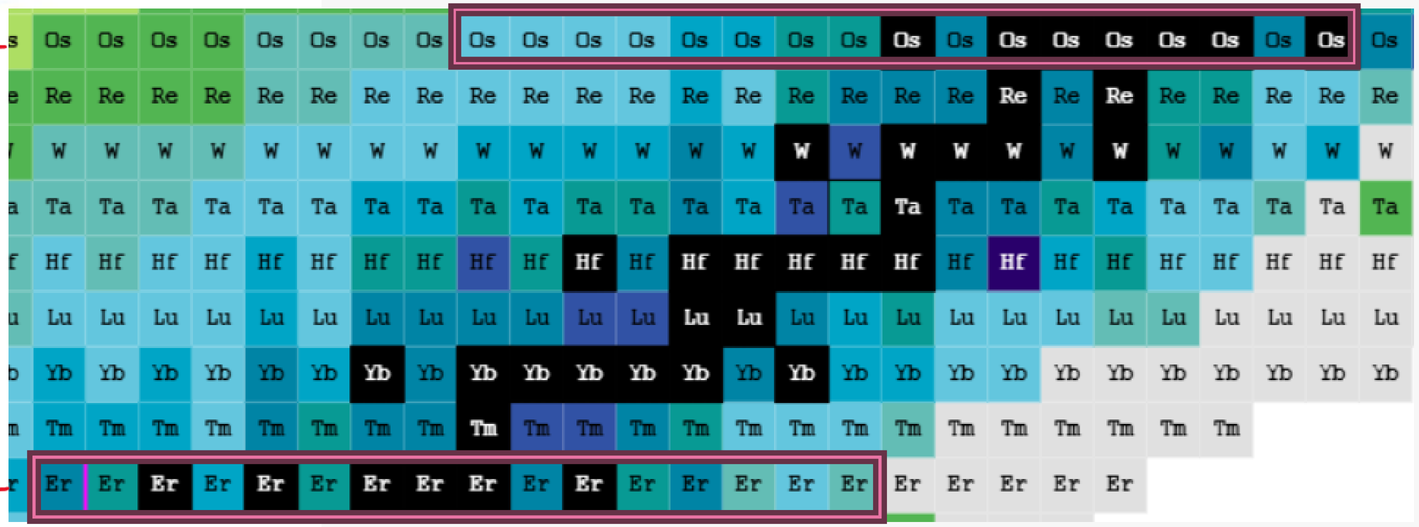
3. Theoretical Results and Comparison with Experimental data
   - Even-even $^{160\text{-}172}Er$
   - Even-even $^{176\text{-}190}Os$

4. Summary and Outlook

Area of Interest & Motivation

- Even-even $^{160\text{-}172}$Er isotopes
- Even-even $^{176\text{-}190}$Os isotopes





- Evolution of deformation between N=82 and N=126 shells.
- Both Er and Os isotopic chains run between these two shells.

Data: *National Nuclear Data Center, http://www.nndc.bnl.gov/*

Area of Interest & Motivation

Motivation

- The isotopes of $_{68}Er$ and $_{76}Os$ exhibit collective properties.
- The challenging nature of 0^+ states of the β-excitation.
- Lack of experimental data for B(E2) transition rates of Er and Os isotopes.
- For the case of ^{172}Er , Gamma-spectroscopy measurement is scheduled at IFIN-HH, Bucharest, Romania. (Yber project)
- Latest measurement of ^{184}Os by our group (NUSTRAP) - under analysis (IFIN-HH).

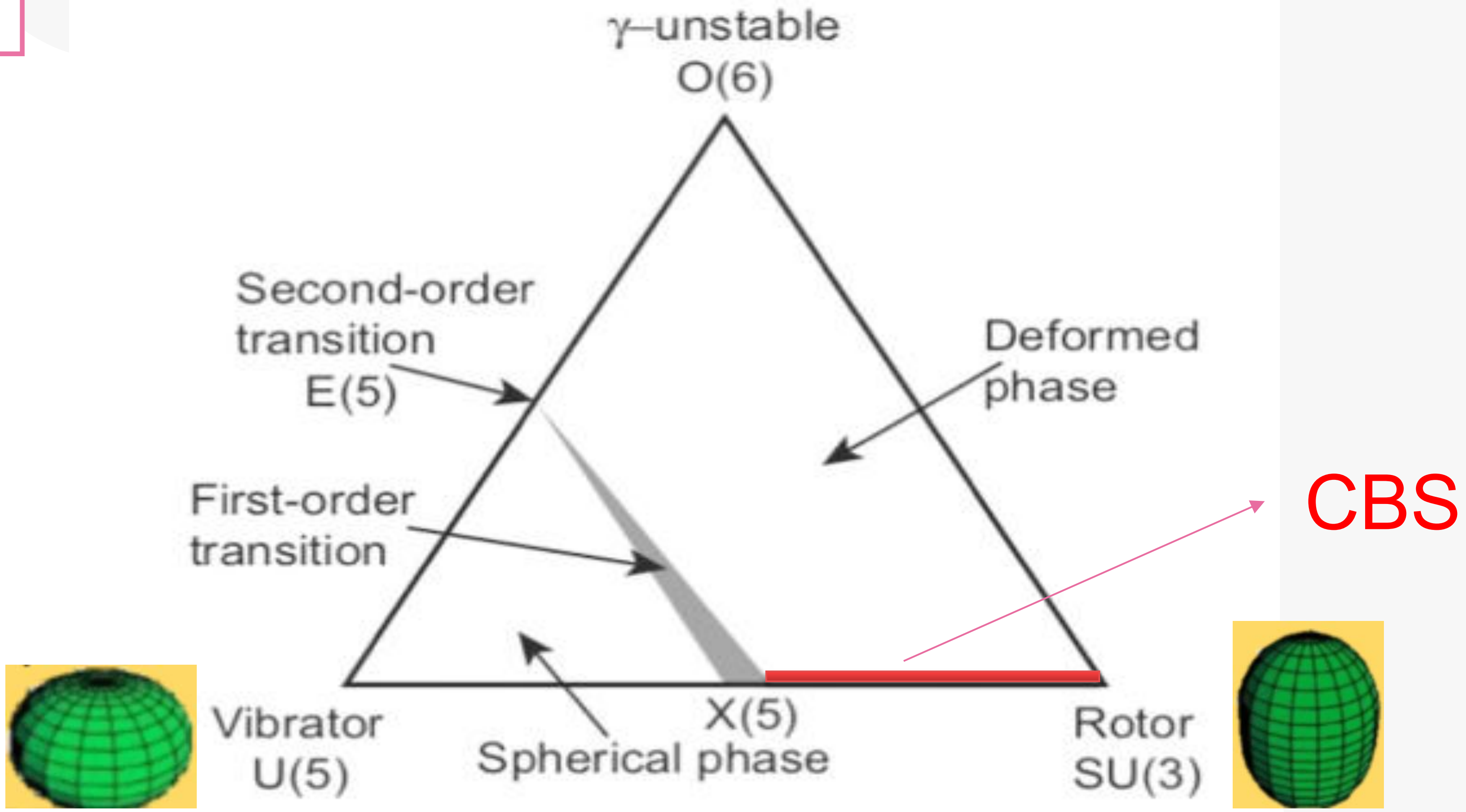
Area of Interest

The IBM-Casten Triangle

The Confined Beta-Soft Rotor Model (CBS) framework bridges the gap between the X(5) critical-point symmetry and the rigid-rotor limit for axially symmetric nuclear shapes.

$\underline{2.9<R_{4/2}<3.333}$

$$R_{4/2}=E_x(4_1^+)/E_x(2_1^+)$$



The model (CBS)

The liquid drop model

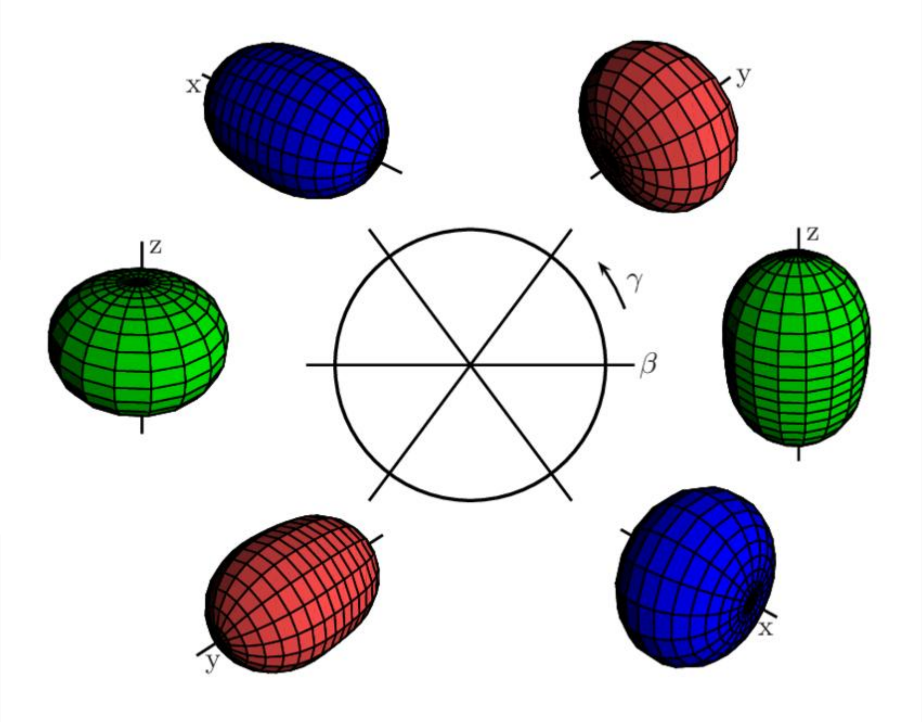
- As the nuclear drop changes its shape and faces deformation, the radius of the drop also transforms with respect to a reference value, R_0, around which oscillations of its surface occur:

$$R(\vartheta,\varphi) = R_0 \left(1 + \sum_{\lambda,\mu} \alpha_{\lambda,\mu} Y_{\lambda,\mu}(\vartheta,\varphi)\right)$$

- Quadrupole deformations (λ=2) dominate these oscillations:

$$R(\theta,\varphi) = R_0 \left[1 + \sum_{\mu=-2}^{2} \alpha^{*}_{2\mu} Y_{2\mu}(\theta,\varphi)\right]$$

- Collective motions in the nucleus — five degrees of freedom.
- The three of them correspond to the orientation of the deformed nucleus (Euler angles Ω(Θ,Φ,Ψ)).
- The other two are the Hill-Wheeler coordinates β and γ defining its shape.



Fortunato, Lorenzo. (2005). European Physical Journal.

The model (CBS)

The generalized Bohr (collective) Hamiltonian:

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_k \frac{Q_k^2}{\sin^2\left(\gamma - \frac{2}{3}\pi k\right)}\right] + V(\beta,\gamma).$$

- We consider potentials $V(\beta,\gamma)=v(\beta)+u(\gamma)$ for axially symmetric prolate ($\gamma\approx 0°$) nuclei.
- Radial part of the Bohr-Hamiltonian:

$$-\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} - \frac{1}{3\beta^2}L(L+1) + u(\beta)\right]\xi_L(\beta) = E\xi_L(\beta)$$

- The wavefunctions can approximately be separated into a radial and an angular part:

$$\boxed{\Psi(\beta,\gamma,\theta_i) = \xi_L(\beta)\,\eta_K(\gamma)\,\mathcal{D}^L_{M,K}(\theta_i)}$$

- We consider an infinite square-well potential

$$U(\beta) = \begin{cases} 0\,, & \beta_m \leq \beta \leq \beta_M\,, \\ \infty\,, & \beta < \beta_m \text{ or } \beta > \beta_M \end{cases}$$

- Now the Schrodinger equation can be solved analytically, becoming a Bessel differential equation:

$$\boxed{\tilde{\xi}''(z) + \frac{1}{z}\tilde{\xi}'(z) + \left[1 - \frac{\nu^2}{z^2}\right]\tilde{\xi}(z) = 0}$$

The model (CBS)

- The general solutions are superpositions of the corresponding Bessel-J and Bessel-Y functions:

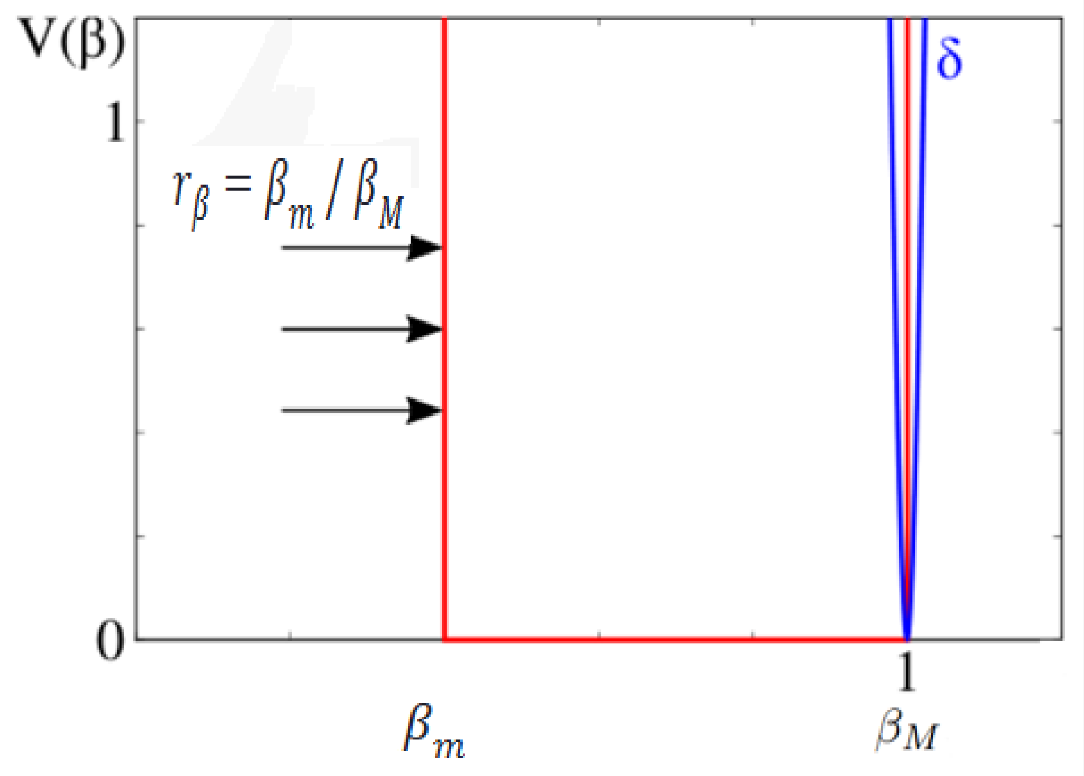
$$\tilde{\xi}_\nu(z) \propto J_\nu(z) + \gamma_Y Y_\nu(z)$$

- <u>The r_β parameter</u>

The infinite square-well potentials forces the wave functions to vanish at both β_m and β_M, leading to a quantization condition:

$$Q_\nu^{r_\beta}(z_M) = J_\nu(z_M)\, Y_\nu(r_\beta z_M) - J_\nu(r_\beta z_M)\, Y_\nu(z_M) = 0$$

where $z_M = \sqrt{E/(\hbar^2/2B)}\,\beta_M$



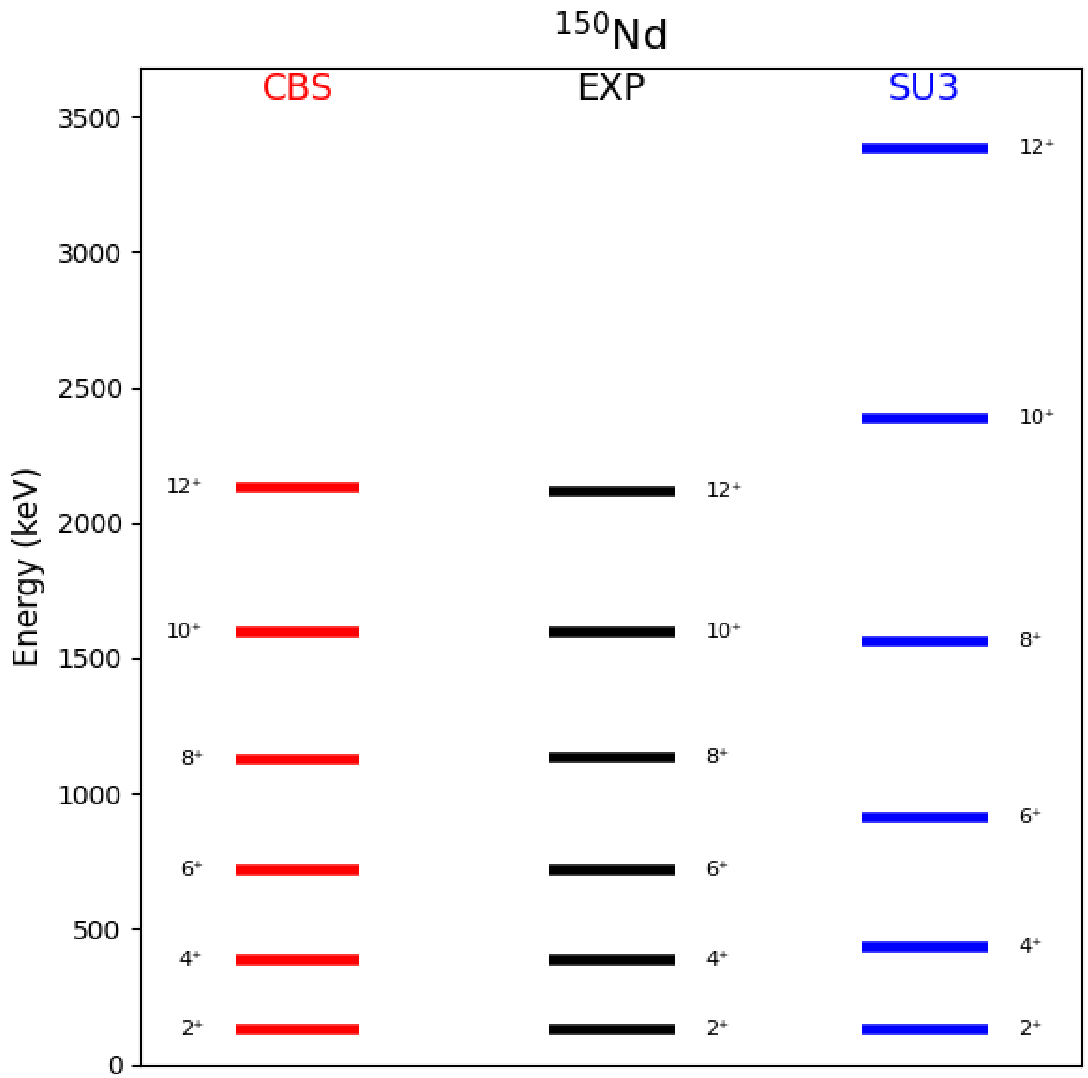
N Pietralla, OM Gorbachenko- Physical Review C–Nuclear Physics, 2004 - APS

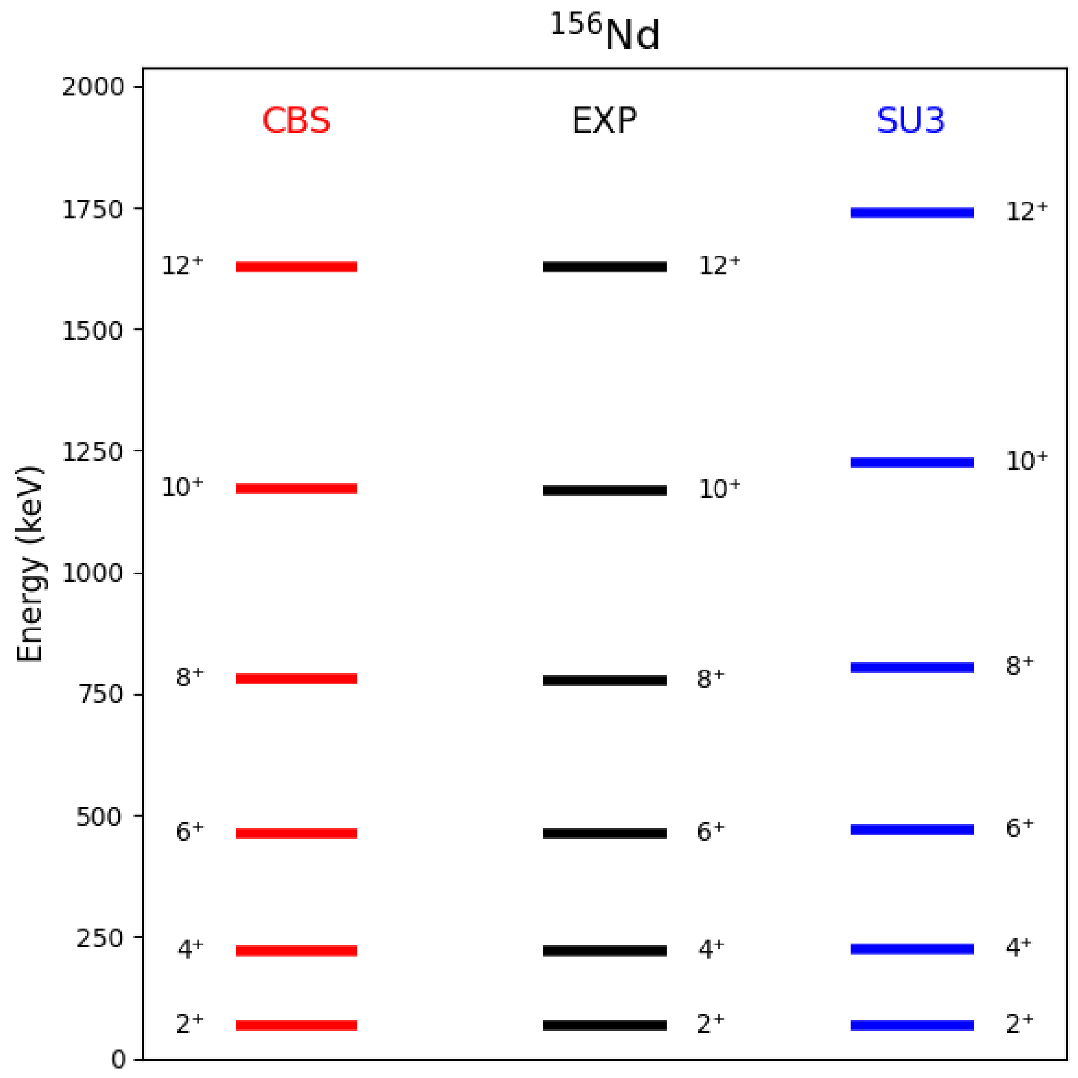
- <u>Finally, we achieve the energy spectrum!</u>

$$\boxed{E_{L,s} = \frac{\hbar^2}{2\,B\,\beta_M^2}\left(z_{L,s}^{\beta}\right)^2}$$

Earlier applications of the CBS

The case of Nd isotopes

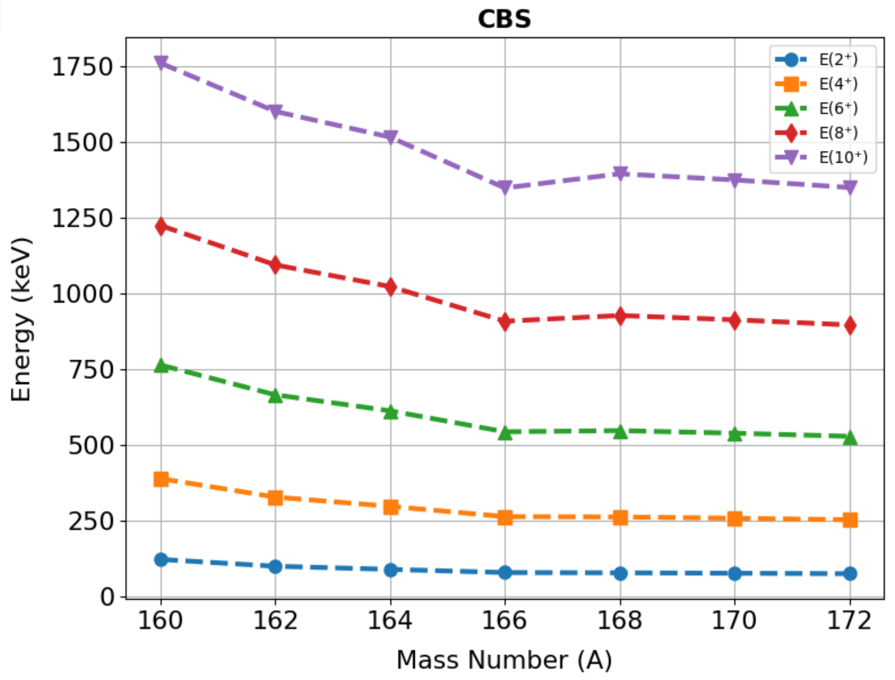


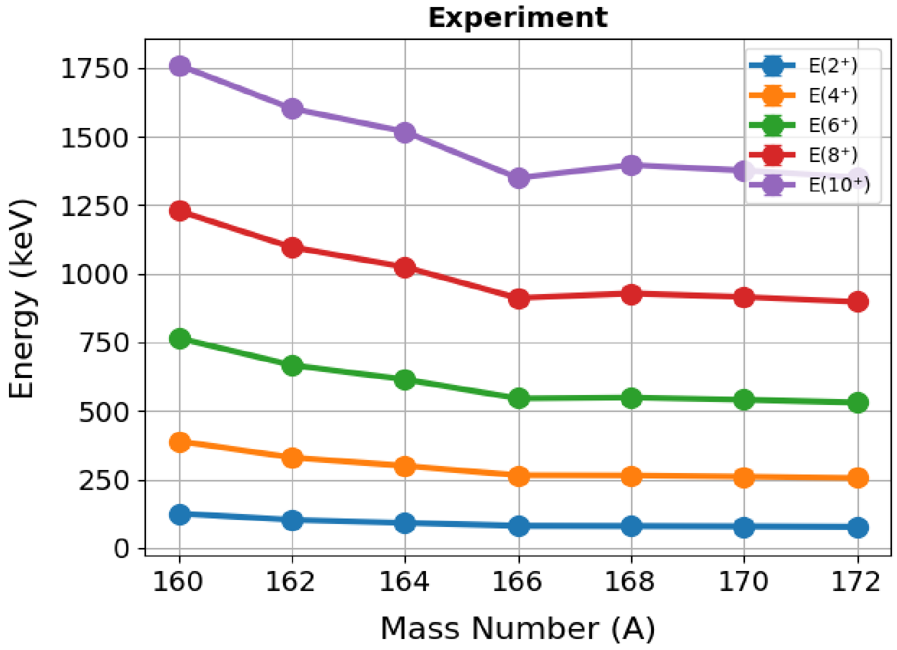


N Pietralla, OM Gorbachenko- Physical Review C—Nuclear Physics, 2004 - APS

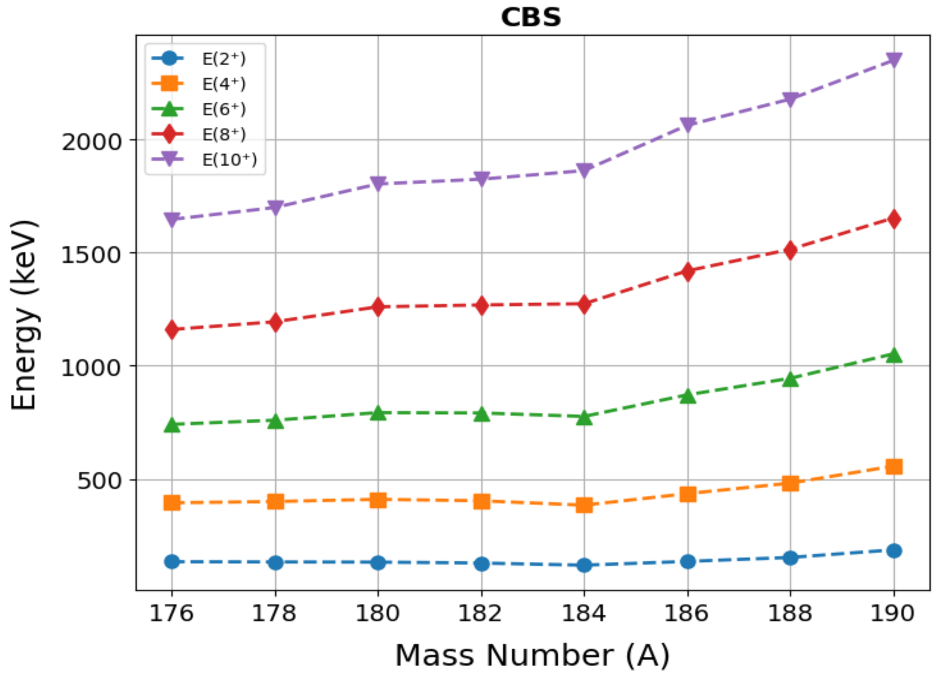
Theoretical (CBS) Results vs Experimental data for even-even $^{160-172}Er$ and $^{176-190}Os$ Isotopes

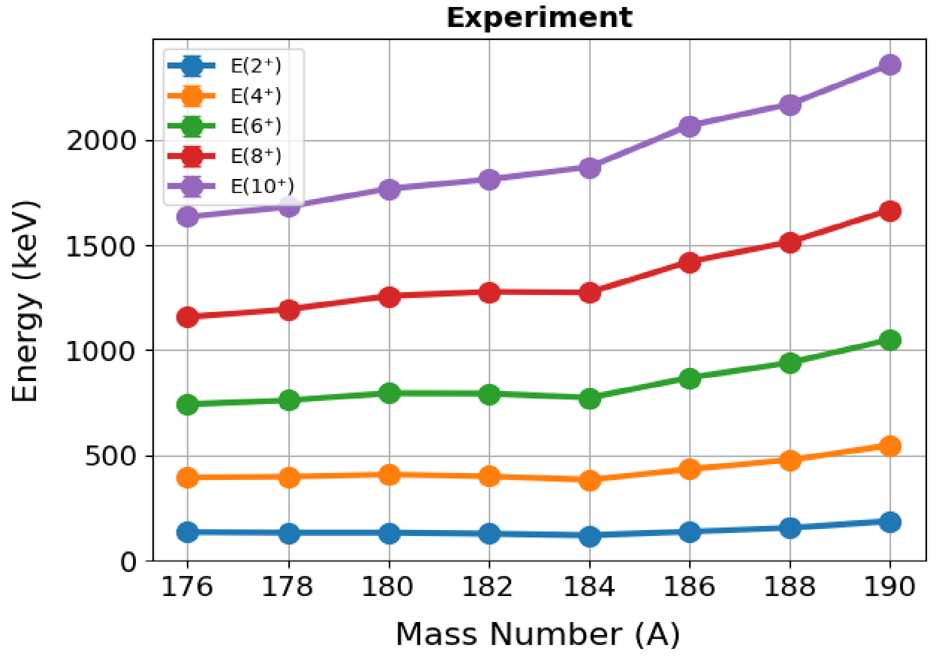
Er isotopic chain (Z=68)





Os isotopic chain (Z=76)





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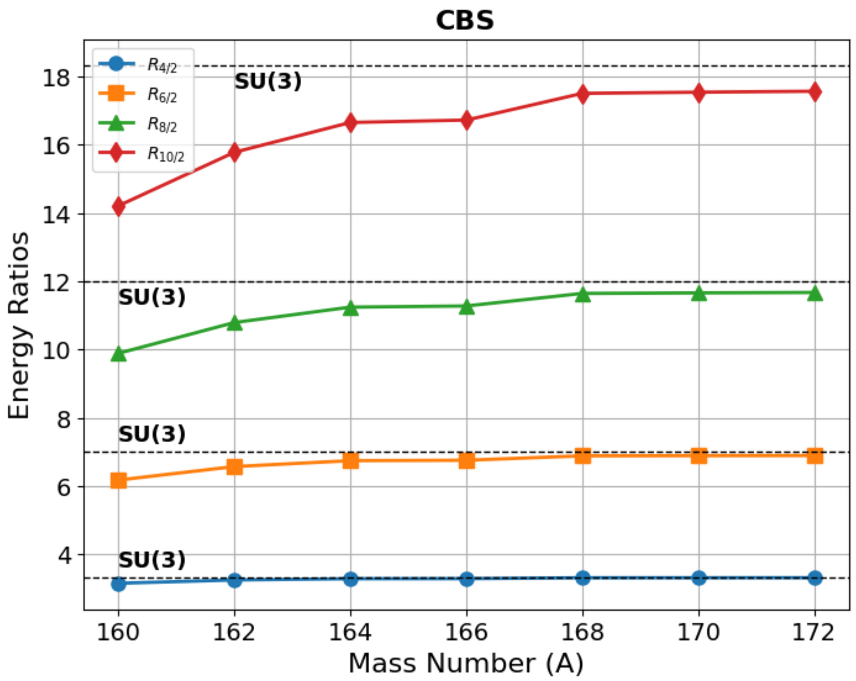

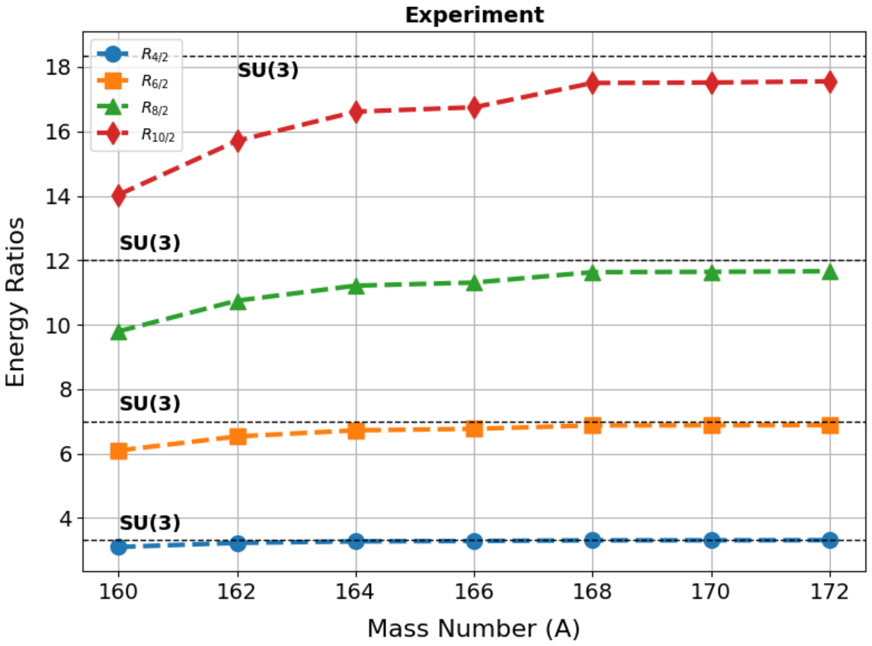

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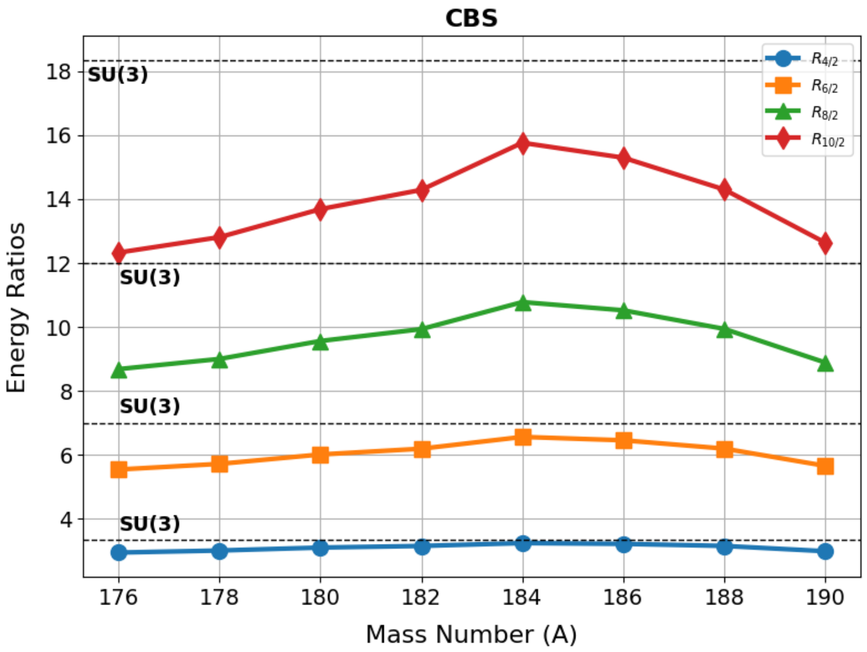
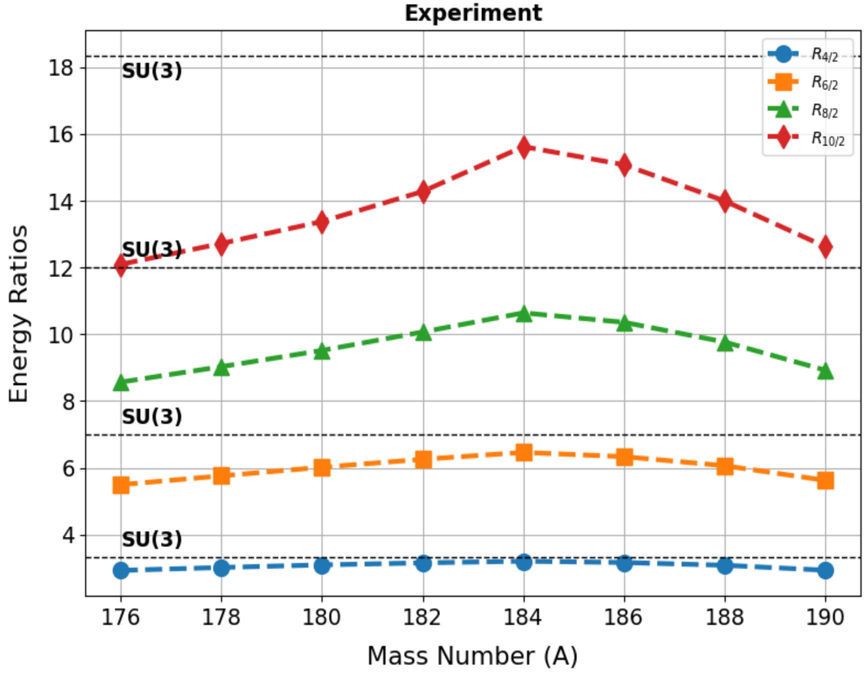
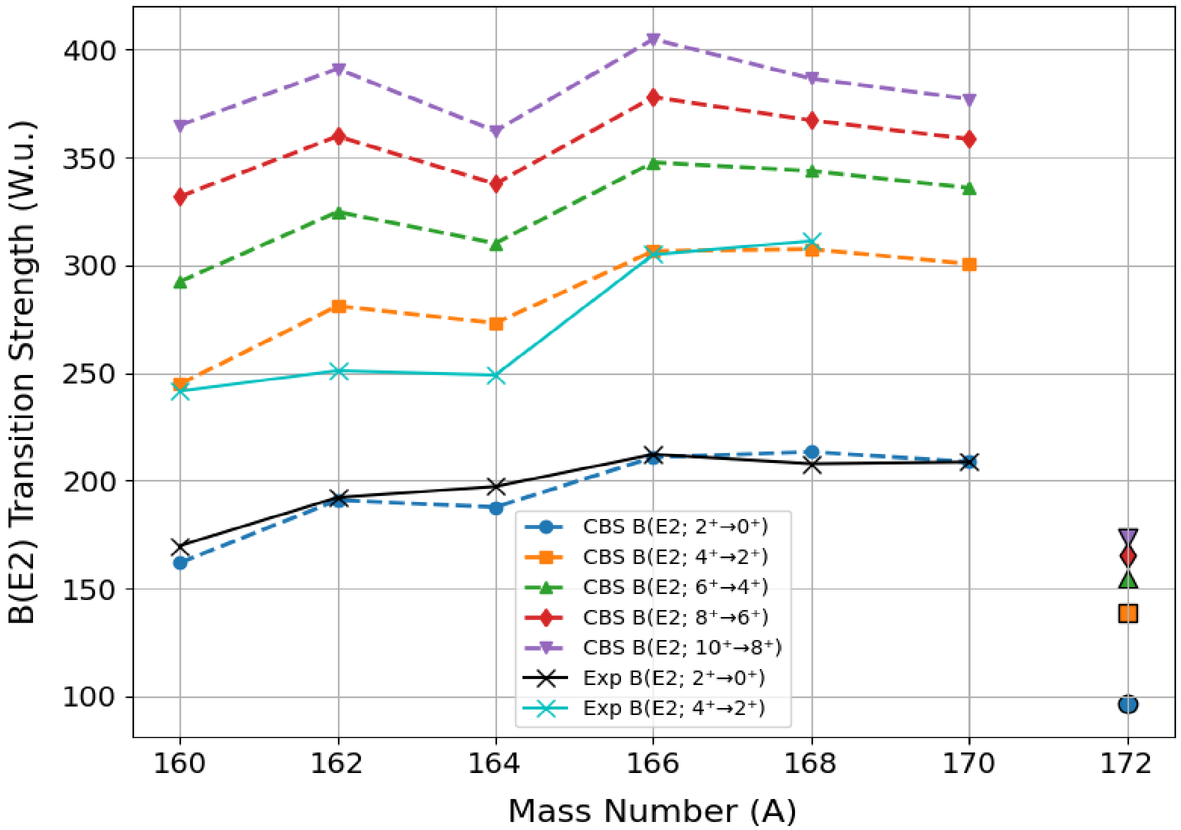


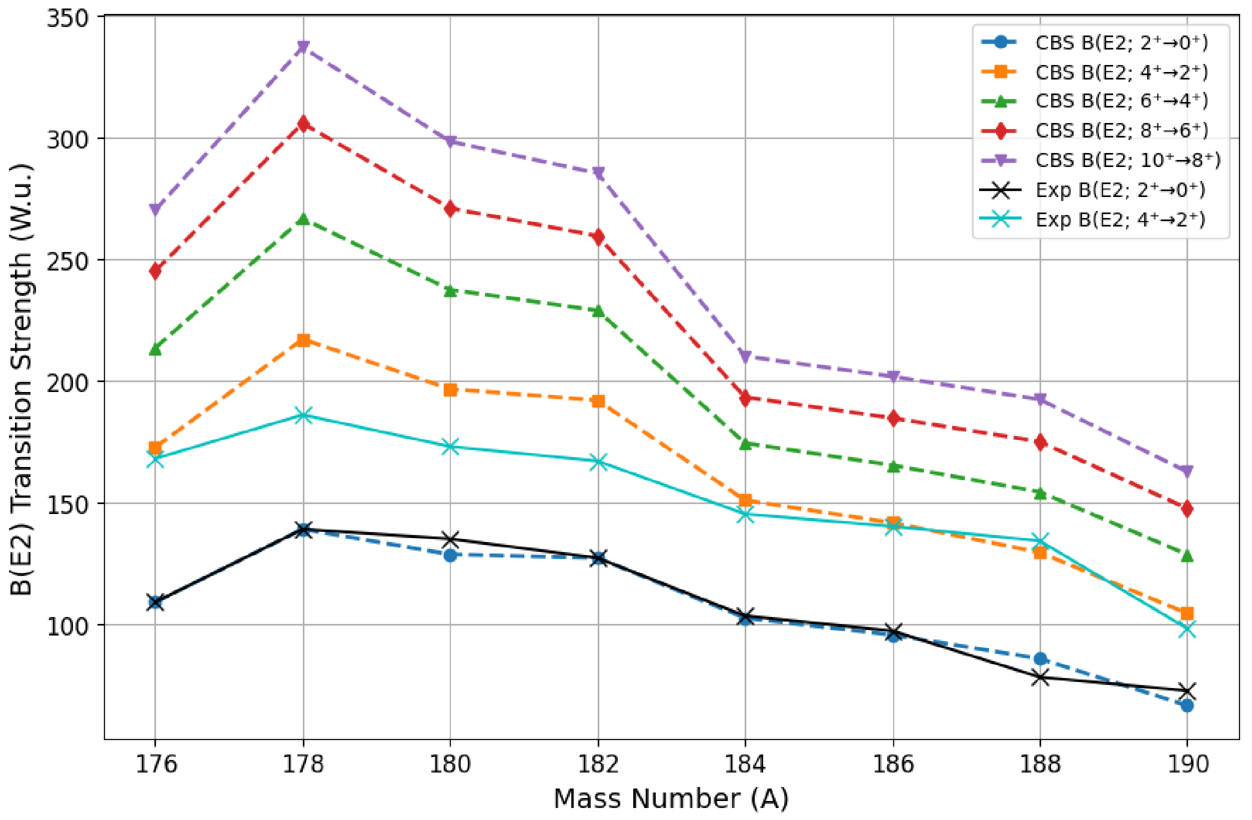


B(E2) transition probabilities of the ground-state band (W.u):

Er isotopic chain (Z=68)

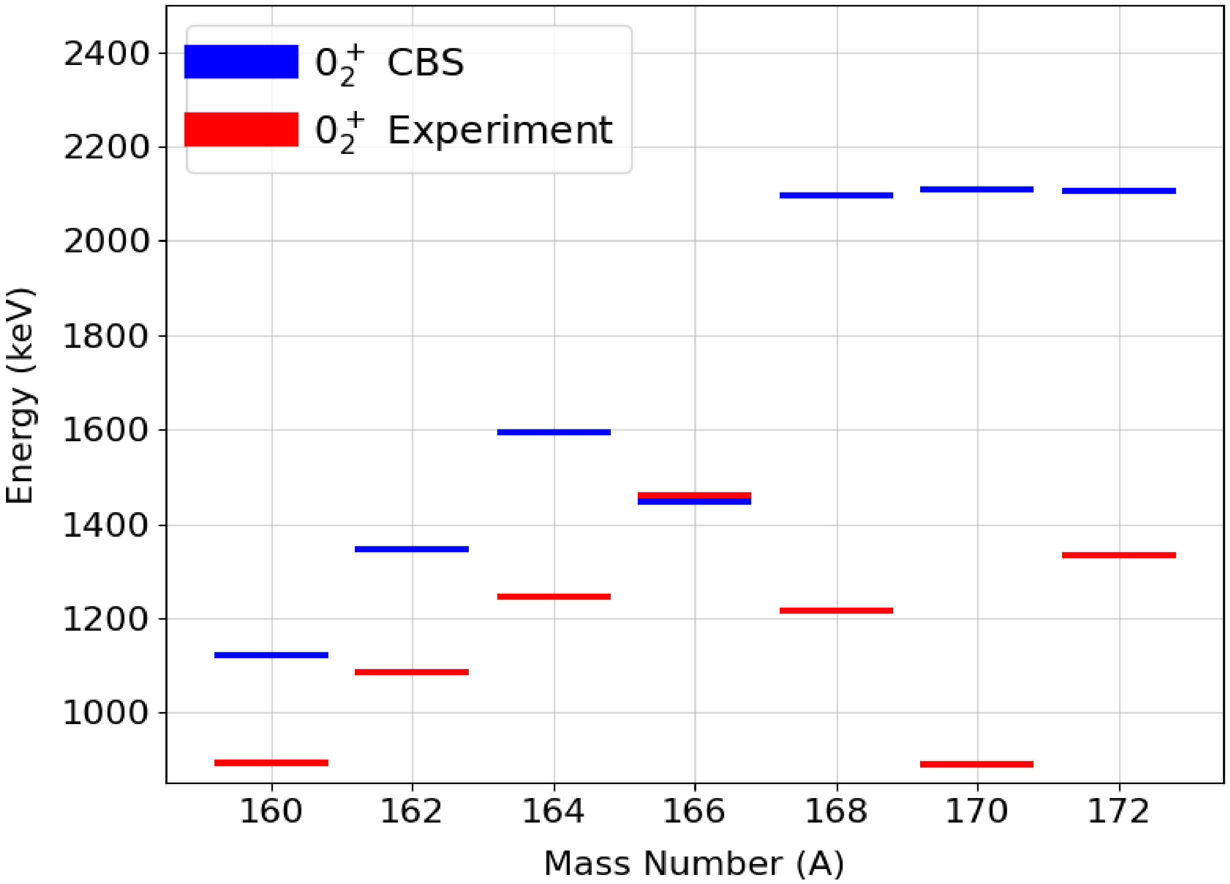


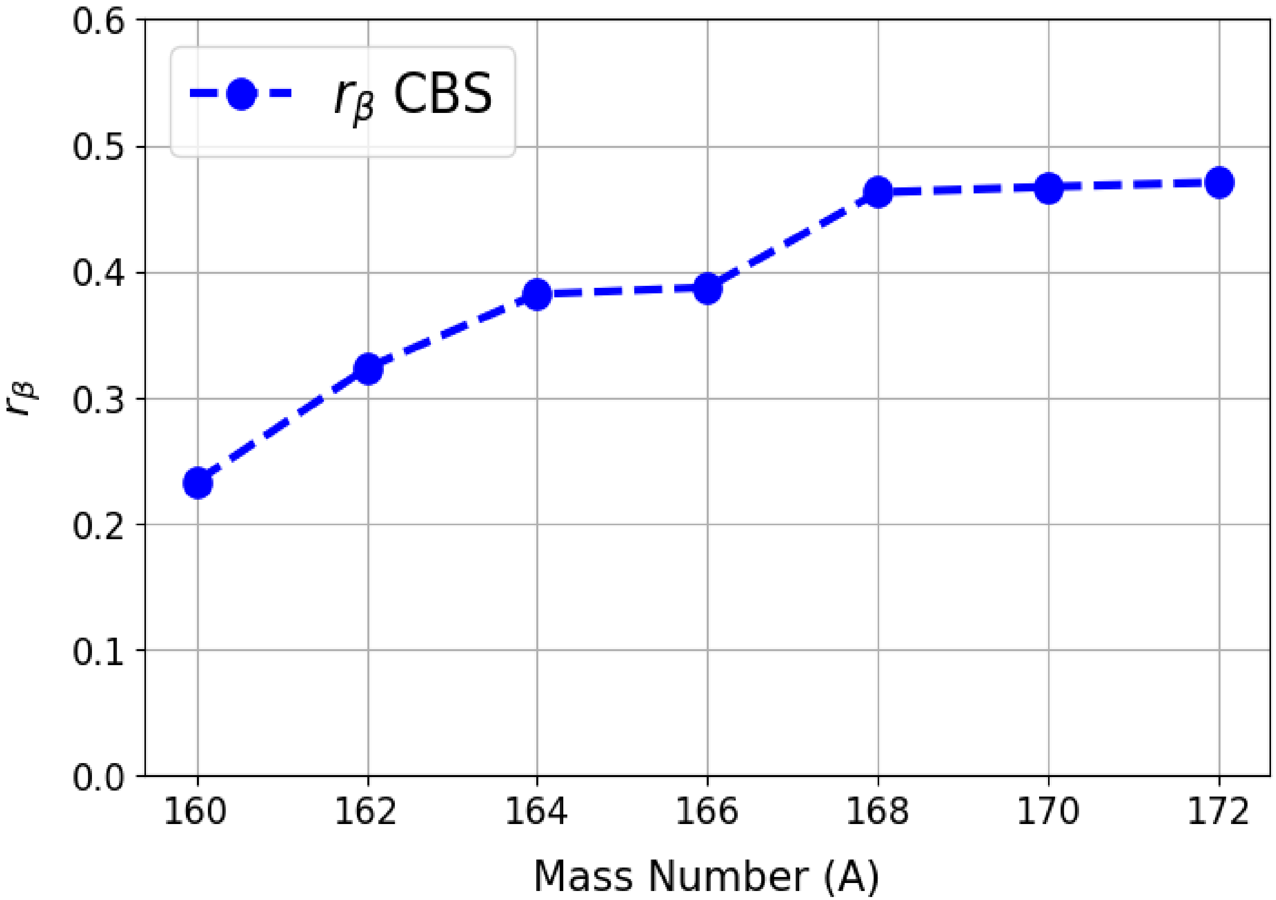
Os isotopic chain (Z=76)



- Lack of experimental data.
- The discrepancy of CBS data is only a matter of scaling, as the $B_{4/2}$ ratio aligns well with the trend of the other isotopes.
- Pre-mid-shell saturation of BE2 values (mid-shell at N=104)

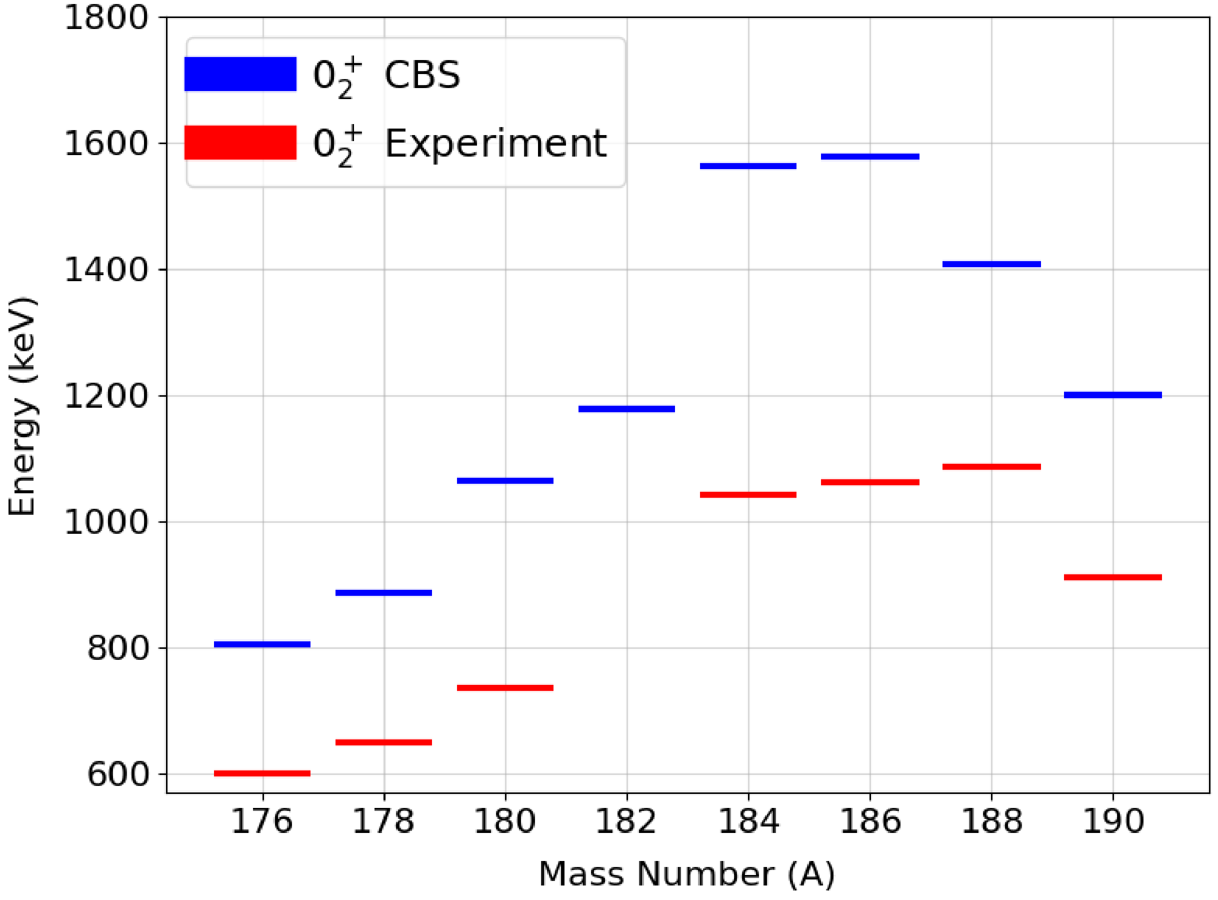
Er isotopic chain (Z=68)

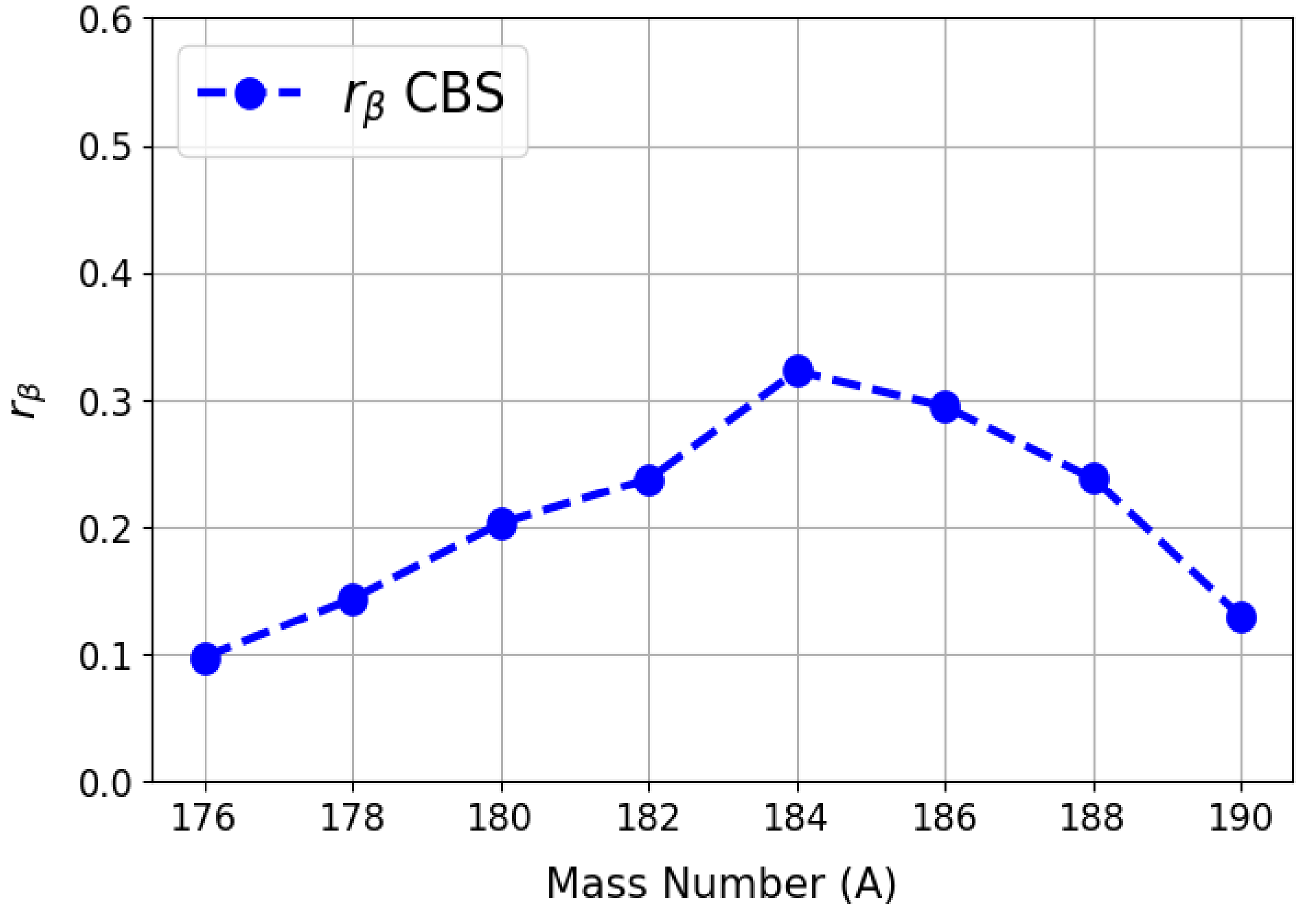




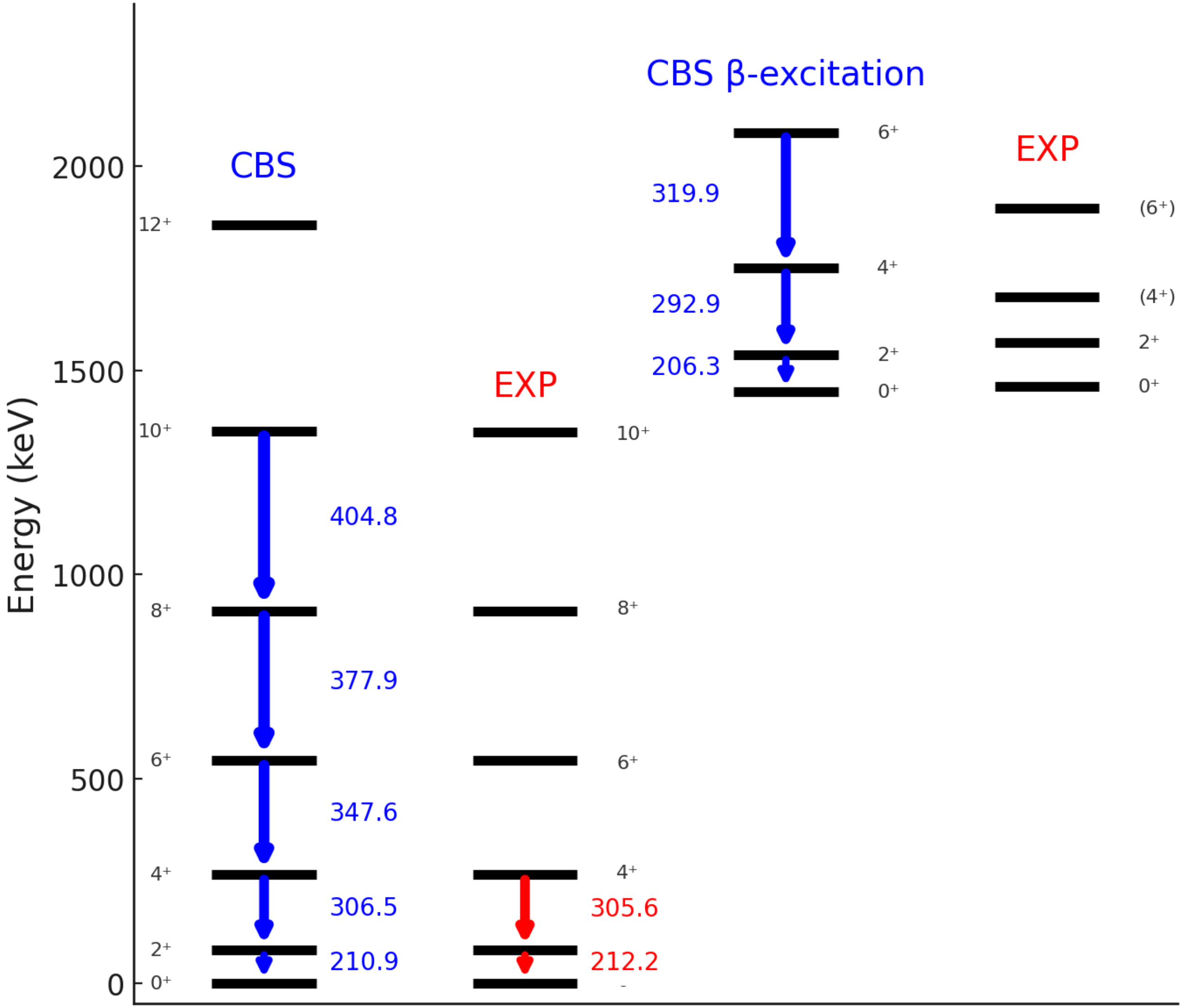
- Approaching the mid-shell, as the potential becomes narrower, the 0^+ states of β-excitation shift to higher energies as mentioned in N. Pietralla et al. Phys. Rev. C 70 (2004) p. 011304 and the nuclei approach the rigid-rotor limit.
- The 0^+ bandheads of CBS usually do not correspond to the first-excited 0^+ states observed, which means that these states are not related to the β-vibration.

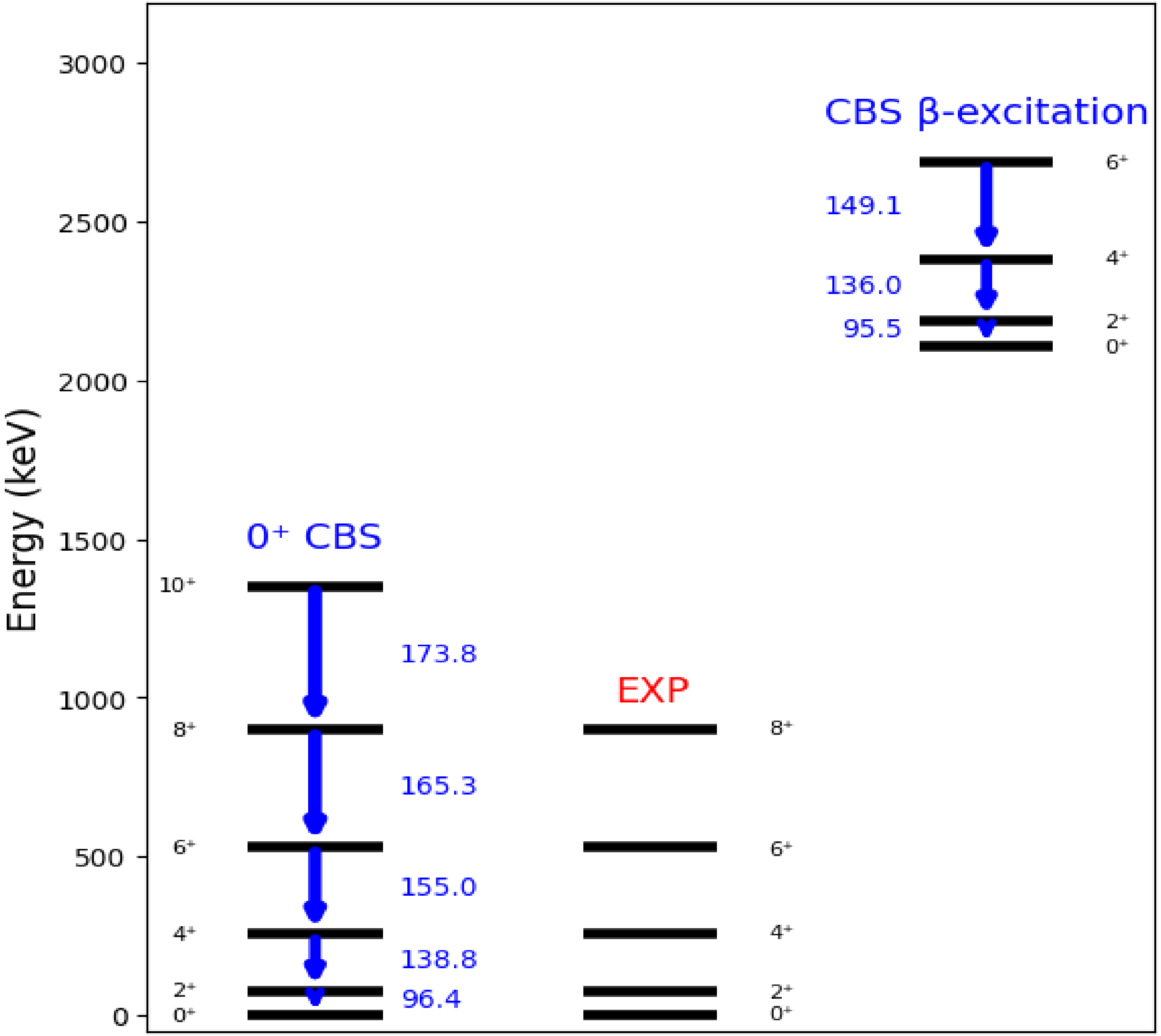
Os isotopic chain (Z=76)





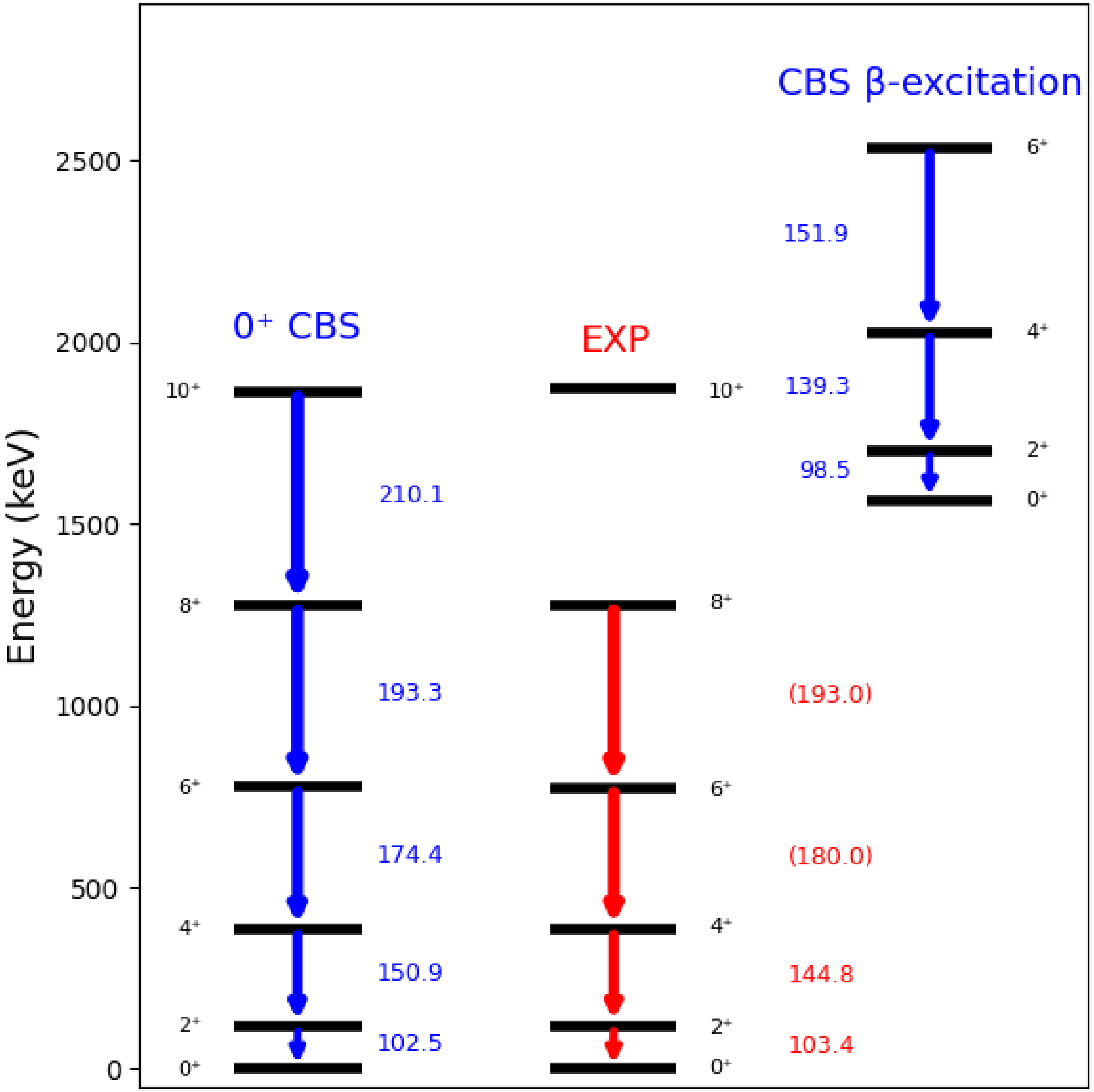
^{166}Er ^{172}Er





- Lack of experimental data of B(E2) transitions to compare.
- For the β-band of ^{172}Er no related band has been reported.

^{184}Os



CBS β-excitation
0⁺ CBS
EXP
Energy (keV)
2500
2000
1500
1000
500
0
10⁺
8⁺
6⁺
4⁺
2⁺
0⁺
210.1
193.3
174.4
150.9
102.5
(193.0)
(180.0)
144.8
103.4
151.9
139.3
98.5

Summary and Outlook

- ✓ Nice agreement between CBS and experimental data for the ground-state band.
- ✓ The 0^+ states of β-excitation proposed provide fresh insight and place tighter constraints on these excitations, moving us closer to a full understanding of the problem.
- ✓ **November 2025**: Gamma-spectroscopy of ^{172}Er *(P. Koseoglou)* at IFIN-HH, Bucharest, search for 0_β^+ and 2_γ^+.
- ✓ We also aim to measure ^{174}Er in the future, moving to more exotic nuclei.
- ✓ As a next step we may assume freedom of γ variable instead of $\gamma\approx0°$ (for example the γ-soft analog of the confined β-soft rotor model), as triaxiality seems to exist in extended regions of the nuclear chart (for example the γ-soft analog of the confined β-soft rotor model).
- ✓ Microscopical approaches, i.e. Ab initio calculations (No core shell-model, Monte Carlo ...).

Thank You

- NuSTRAP group in NKUA

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Walter Benjamin-Programm
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